THE DOUBLE ELECTRICAL LAYER AT THE SURFACE OF A SATELLITE

By

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As is known, a satellite acquires a certain charge in the ionosphere, and accordingly a double electrical layer is formed at its surface [1, 2] (Fig. 1). Knowledge of the structure and properties of the double electrical layer is very essential: the interaction between the satellite and the ionosphere occurs "through" the double layer; the double layer determines the boundary conditions, and also affects the physical agents of the more complicated processes of ionospheric disturbance, the electron and ion beams formed by the satellite. Three equations describing the distribution of the electrical field in the double layer have been given in the literature. These equations differ, however, and lead to differing dependences of the potential upon the distance from the wall. It is advantageous, therefore, to indicate briefly the differences between these three theories and what they give when applied to a concrete problem of a satellite in the ionosphere. This is the purpose of the present review [January, 1959].

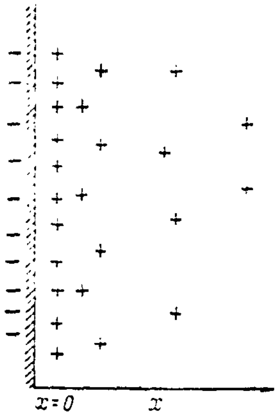


Fig. 1.

1. The Double Layer in Thermodynamic Equilibrium [3]

The original position of the classical theory of the double layer used in electrochemistry and colloid chemistry is that charged particules (ions) are in the double layer in a state of thermodynamic equilibrium, and therefore, are described by the Maxwell-Boltzmann distribution function

$$F(u, x) = Ce^{-\frac{mu^2}{2kT} - \frac{e\varphi}{kT}}, \tag{1}$$

where u is velocity, x a coordinate, $\varphi = \varphi(x)$ the potential, e the charge of a particle, m its mass, and T temperature. For clarity, let us examine only the one-dimensional case (a plane double layer, Fig. 1) when the problem is solved in quadratures. The other cases (spherical, cylindrical layer) are solved by numerical integration of the corresponding equations, but the physical picture remains the same as for a plane double layer. If the thickness of the double layer d is much less than the linear dimensions of the surface of the satellite L_c ($d \ll L_c$ for large satellites with $L_c \approx 1$ m), then the double layer can be assumed plane in the first approximation. Therefore, the plane double layer is an important limiting case, by which it is convenient to explain the physical picture.

The kinetic equation

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} - \frac{e}{m}\frac{\partial \varphi}{\partial x}\frac{\partial F}{\partial u} = 0 \qquad (2)$$

has, as is easily verified by substitution, a standard solution

$$F = \chi\left(\frac{mu^2}{2} + e\varphi\right), \qquad (3)$$

where χ is an arbitrary function of its argument $\frac{mu^2}{2} + e\varphi$. Obviously, the form of the function χ is determined by the boundary conditions. In our case there are two such conditions: $x = 0$ and $x = \infty$. At $x = 0$, as in all space, χ must be a symmetric function of velocity (i.e., at any u, $\chi(u) = \chi(-u)$); the wall is reflecting. At infinity ($\varphi = 0$), the function χ must convert to Maxwell distribution (1). By choosing χ in the form of (1), we satisfy both boundary conditions and obtain, there, a unique solution of the problem of a double layer in equilibrium near a reflecting wall.

The theorem that electrons and ions in the double layer near a reflecting wall are described by distribution (1) follows from the general principles of statistical physics. This theorem is examined in a work by Mott-Smith and Langmuir [4]. It is valid for any sign of the satellite potential φ_c, for a decelerating as well as for an accelerating field.

By knowing the distribution function (e.g., (1)) for ions and electrons, it is easy to calculate the field in the double layer. The Poisson equation in this one-dimensional case takes the simple form

$$\frac{d^2\varphi}{dx^2} = +4\pi n_0\varepsilon\left(e^{\frac{\varepsilon\varphi}{kT_e}} - e^{\frac{-\varepsilon\varphi}{kT_i}}\right), \qquad (4)$$

where ε is the absolute value of the electron charge, n_0 the electron and ion concentration at infinity (in an undisturbed ionosphere), and $T_e = T_i$ is the electron and ion temperature.

Introducing the new variables

$$y = \frac{\varepsilon\varphi}{kT} \text{ and } \xi = \frac{x}{R_D}\sqrt{2}\left(R_D = \sqrt{\frac{kT}{8\pi n_0\varepsilon^2}} \text{ is the Debye radius}\right) \quad (5)$$

let us transform (4) to

$$y'' = \sinh y \quad (6)$$

(the double prime denotes differentiation with respect to ξ). Let us multiply both sides of (6) by $2y'$:

$$2y'y'' = 2 \sinh y y'.$$

The derivative of $(y')^2$ is on the left, the derivative of $2 \cosh y$ on the right, i.e., $(y')^2 = 2 \cosh y + C_1$.

When $\xi \to \infty$, $y = 0$, so that

$$C_1 = -2$$

and

$$y' = -2 \sinh \frac{y}{2} \quad (7)$$

Let us integrate again:

$$-\xi = \int \frac{dy}{2 \sinh \frac{y}{2}} + \ln C_2 = 2\ln \frac{e^{y/2}+1}{1-e^{y/2}} + \ln C_2. \quad (8)$$

When $\xi = 0$, $y = \varepsilon\varphi_c/kT$ (φ_c is the potential of the satellite). Let us denote this boundary value by $z (z = \varepsilon\varphi_c/kT)$. Then

$$C_2 = \frac{1+e^{z/2}}{1-e^{z/2}}$$

and finally

$$\varphi = \frac{kT}{\varepsilon} 2\ln \frac{e^{z/2}(1+e^{-\xi}) + (1-e^{-\xi})}{e^{z/2}(1-e^{-\xi}) + (1+e^{-\xi})}. \quad (9)$$

The double layer can be divided into two regions. In the first region near the wall $|y| = \varepsilon|\varphi|/kT > 1$, i.e., the potential of an electron or ion in the field of the layer is greater than the energy of its thermal motion. In this region a space charge is created by ions, and the electron concentration is exponentially small ($n_e \sim n_0 e^{-|y|}$). Equation (4) takes the form: $y'' = e^y$, i.e., it is essentially nonlinear.

In the second region, at $|y| << 1$, the potential energy can be considered a small addition to the energy of thermal motion. Correspondingly, Eq. (4) becomes linear: $y'' = y$ (when $|y| << 1$, $\sinh y \approx y$). A space charge $\rho = n_1 - n_e$ is created by ions and electrons (the concentration of both is small):

$$\frac{n_i - n_0}{n_0} \ll 1, \quad \frac{n_e - n_0}{n_0} \ll 1.$$

The potential drops in this region according to the law $\varphi = \varphi_0 e^{-\xi}$.

The difference between the linear and nonlinear regions of the double layer is shown in Fig. 2, but not to scale.

The dependences of the potential φ, field strength E, and space charge ρ upon the distance to the wall x which follow from Eq. (4) and its solution (9) are shown in Figs. 3 to 9 (smooth curves) for various values of the potential of the satellite φ_c = -0.77 v (Figs. 3 to 5), φ_c = -6 v (Figs. 6 to 8), and φ_c= - 30 v (Fig. 9). The potential and field drop vary rapidly: at a distance $x = R_D$ even at low satellite potentials (φ_c = -0.77 v, z = -5.15) the potential decreases to 8% φ_c, the field to 7% of its value at the wall, and the ion concentration to 0.5% of its value at the wall. At φ_c = -6 v, this decrease goes even more rapidly. Thus according to this theory, we can use the Debye radius R_D as the thickness of the double layer d. Let us estimate its maximum value for altitudes of 300 to 400 km. According to the literature [5], at night $n \approx 10^5/cm^3$, $kT \approx 0.15$ ev, then R_D = 1 cm (maximum thickness of the double layer).

For a double layer at equilibrium,(4) through (7), the field strength at the wall increases exponentially with an increase (modulo) in the satellite potential φ_c, reaching 47 kv/cm at φ_c = -3 v (z = = -20). It is namely this singularity, together with the dependence upon φ_c of the capacitance of the double layer (see Section 3), which could serve for comparison of theory with experiment, and for selection of a correct model of the double layer by independent measurement of E and φ_c. In this it should be borne in mind that, due to unevenness of the surface of the satellite, E is measured not at $x = 0$, but at some distance from the wall on the order of 0.01 to 0.02 mm, so that the experimental value of $E(x = 0)$ is close to the average value of E at $x = 0.001$ to 0.002 cm. At $z = -20$, $x = 0.002$ cm, $R_D = 0.2$ cm, by formula (7) we find $E = 300$ v/cm, which is considerably less than 47 kv/cm.

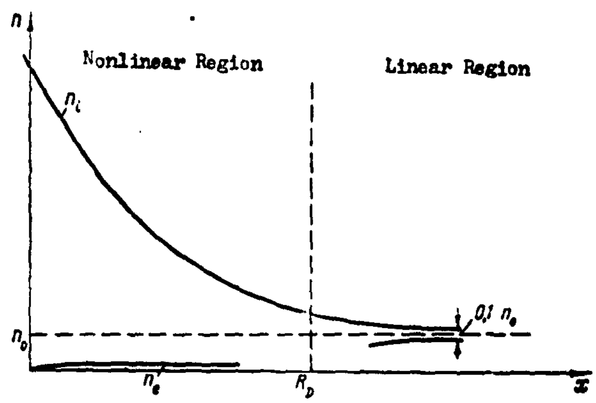


Fig. 2.

2. The Langmuir-Bohm Equation [6, 7]

As before, the electron distribution is assumed to be a Boltzmann distribution, i.e.,

$$n_e = n_0 e^{e\varphi/kT}.$$

Under our conditions, when $\varphi_c < 0$ and $z > 1$, almost all electrons are reflected, and this assumption is valid.

However, the ion distribution takes another (non-Boltzmann) form. Ignoring the thermal energy of the ions, Langmuir and Bohm took

$$\frac{mu^2}{2} + e\varphi = \text{const} = 0. \quad (10)$$

From the condition of constancy of conduction current

$$\operatorname{div} j = 0$$

we find the relationship between the concentration of ions and their velocity at a given point:

$$j = n\sqrt{\frac{2\varepsilon|\varphi|}{M}} = n_0\sqrt{\frac{2\varepsilon|\varphi_0|}{M}},$$
$$n = n_0\sqrt{\frac{\varphi_0}{\varphi}}, \tag{11}$$

i.e., approaching a negatively charged wall, the concentration of positive ions does not increase, as in a double layer in equilibrium, but, conversely, drops. This reasoning is also applicable for an absorbing wall. Substituting (11) into the Poisson equation $\frac{d^2\varphi}{dx^2} = -4\pi\varepsilon(n_i - n_e)$, we find the Langmuir-Bohm equation of the double layer

$$\frac{d^2\varphi}{dx^2} = -4\pi\varepsilon n_0\left[\sqrt{\frac{\varphi_0}{\varphi}} - e^{\frac{\varepsilon(\varphi-\varphi_0)}{kT}}\right]. \tag{12}$$

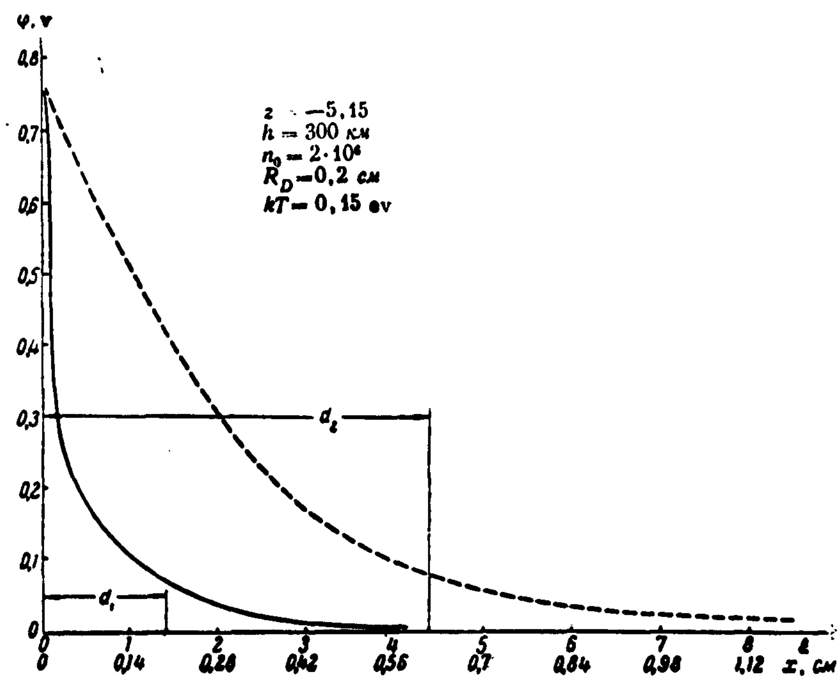


Fig. 3.

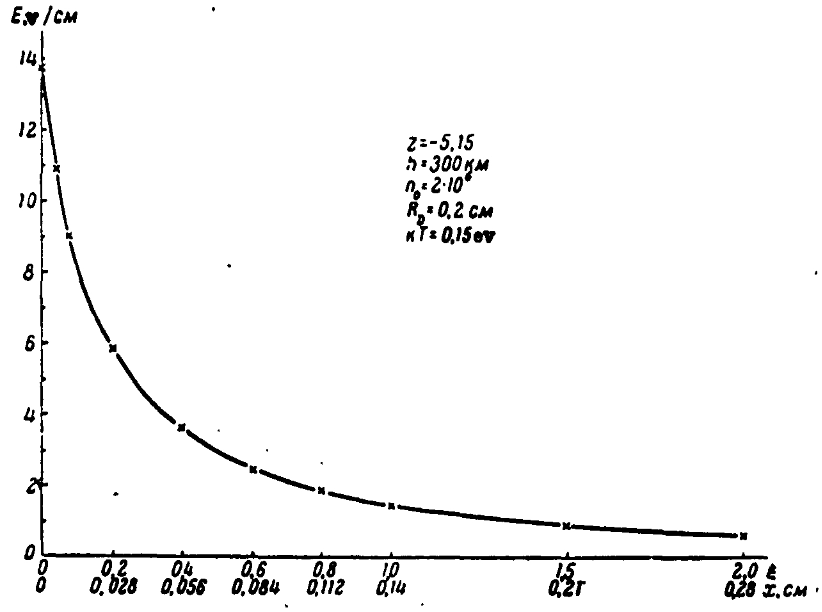

E,v/cm
14
12
10
8
6
4
2
0
z=-5,15
h=300 km
n_0=2·10⁶
R_0=0,2 cm
kT=0,15 ev
0 0,2 0,4 0,6 0,8 1,0 1,5 2,0 ξ
0 0,028 0,056 0,084 0,112 0,14 0,21 0,28 x, cm

Fig. 4.

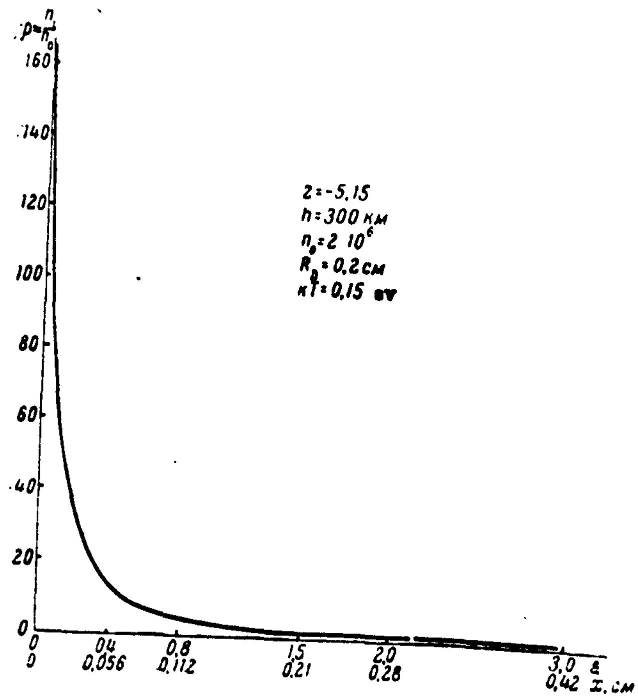
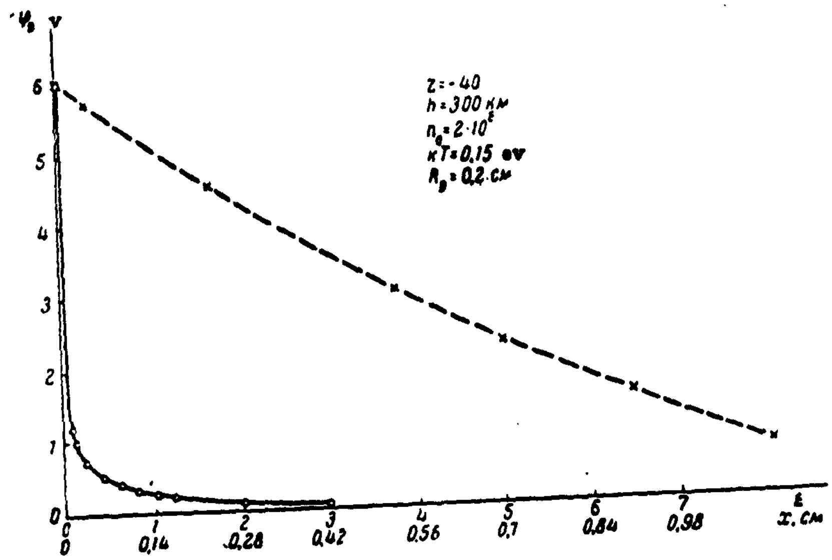
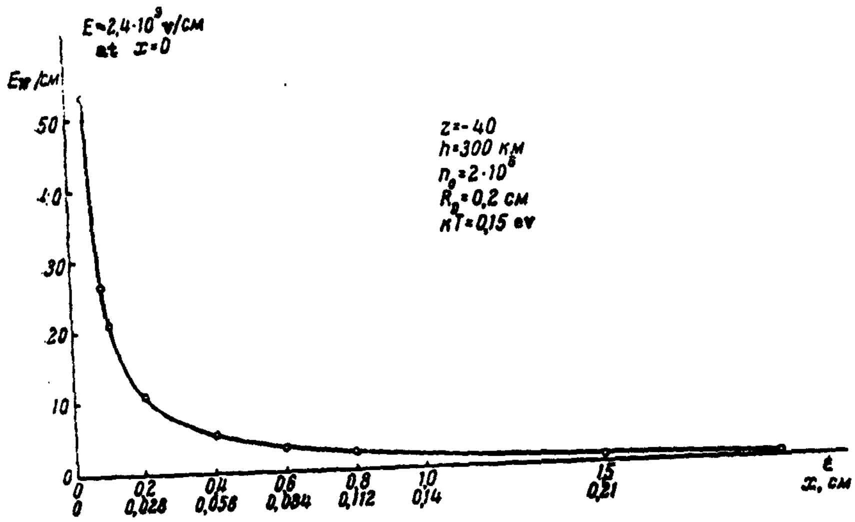

p=n/n₀
160
140
120
100
80
60
40
20
0
z=-5,15
h=300 km
n_0=2 10⁶
R_0=0,2 cm
kT=0,15 ev
0 0,4 0,8 1,5 2,0 3,0 ξ
0 0,056 0,112 0,21 0,28 0,42 x, cm

Fig. 5:



φ₀ v
z = -40
h = 300 км
n₀ = 2·10⁶
κT = 0,15 ev
R₀ = 0,2 см
ξ
x, см

Fig. 6.



E = 2,4·10⁹ v/см
at x = 0
Ev/см
z = -40
h = 300 км
n₀ = 2·10⁶
R₀ = 0,2 см
κT = 0,15 ev
ξ
x, см

Fig. 7.

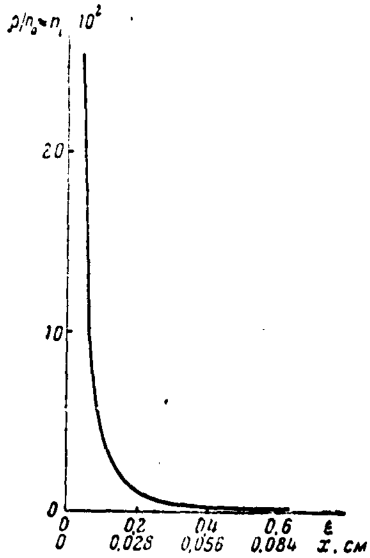


Fig. 8.

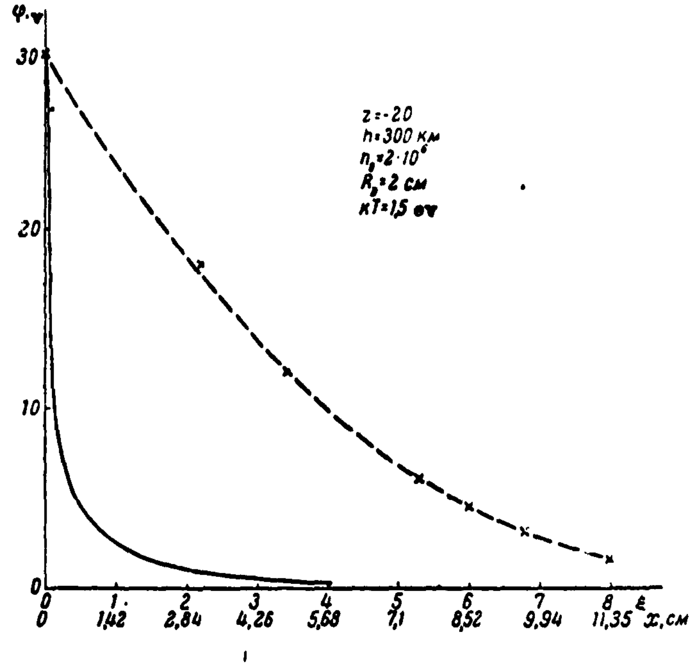


Fig. 9.

Now a fundamental difficulty arises: we cannot find strict boundary conditions for this equation. If it is assumed that at infinity $n = n_0$, as in Section 1, then, because $\varphi \rightarrow 0$ as $x \rightarrow \infty$, ions in the double layer will have infinite velocity, and their concentration will be zero. Assigning a certain condition $u = u_0$ when $\varphi = \varphi_0$ at a finite distance from the wall, we, in essence, introduce the same value which we excluded by relation (10), i.e., thermal motion. To be consistent, we cannot let $u = u_0$ at $x = x_0$ when taking thermal motion into account. Instead of $u = u_0$, we must introduce the distribution function $f(u, x)$, and also take the velocity spread into account. Langmuir and Bohm set the boundary condition approximately: they took the field strength $E = -d\varphi/dx = 0$ at the boundary of the double layer at $\varphi = \varphi_0 \sim \frac{kT}{e}$. (Strictly speaking, $E = 0$ when $\varphi = 0$, i.e., at infinity.)

Multiplying (12) by $d\varphi/dx$, let us integrate and use the approximate boundary condition

$$\text{when} \quad x = x_0 \quad \varphi = \varphi_0, \quad \frac{d\varphi}{dx} = 0. \tag{13}$$

We obtain the following equation for φ:

$$\left(\frac{d\varphi}{dx}\right)^2 = +8\pi n_0 \varepsilon \left\{2\varphi_0 \left(\sqrt{\frac{\varphi}{\varphi_0}} - 1\right) + \frac{kT}{\varepsilon}\left(e^{\frac{\varepsilon(\varphi-\varphi_0)}{kT}} - 1\right)\right\}. \tag{14}$$

This equation cannot be integrated in quadratures. It is integrated numerically [7], where it is taken that $\varphi_0 \sim kT/e$ (the double layer ends where the energy of thermal motion is of the same order as the potential energy in the field of the double layer).

Qualitatively, the nature of the solutions of Eq. (14) can be explained as folows. This equation is more accurate, the lower x. But when $x \ll R_D$, $|\varepsilon(\varphi - \varphi_0)|/kT > 1$, and the exponential term in Eq. (12) is negligible in comparison with $\sqrt{\varphi_0/\varphi}$. Then

$$\frac{d\varphi}{dx} = 4\sqrt{\pi n_0 \varepsilon}\sqrt[4]{|\varphi|}, \text{ i.e., } |\varphi|^{3/4} = |\varphi_c|^{3/4} - 3\sqrt{\pi n_0 \varepsilon}x, \tag{15}$$

i.e., $|\varphi|$ decreases somewhat more rapidly than according to the linear law $\varphi = \varphi_c + ax$. If there were a total absence of electrons and ions at the wall, then the potential would be a linear function of x, and the field strength would be constant. The presence of ions leads to a decrease in the field in the double layer. But, of course, the field decreases much more slowly than according to law (9), and, accordingly, the double layer is much thicker. For example, at $z = -26$, φ decreases to $0.1\varphi_c$ at $x \approx 100R_D$ [7], while according to (9) this same relative drop in potential occurs at $x = 0.2R_D$ (Fig. 3, smooth curve). Such a slow decrease in the field would lead in the ionosphere to double layers with a thickness of about one meter, which is highly improbable.

An advantage to Eq. (12) is that it does not require an ideally reflecting wall and thermodynamic equilibrium. However, this equation does not take thermal motion into account.

All of the above pertains to a wall which is fixed relative to the plasma. Now let us estimate the set-up time of the double layer τ_i and compare it with the time interval τ_c during which the field of the satellite is at a given point in space. The condition for setting up a double layer is $\tau_c \gg \tau_i$. Distribution (1) or (11) is set up, roughly speaking, during time τ_c, the time necessary for an ion to fly through the double layer. Let us find the upper limit $\tau_i = \tau_{max}$. Let us assume that an ion is uniformly accelerated; then $\tau = \sqrt{2d/a}$, where $a = \varepsilon E/m$ is the acceleration. Calculating τ_{max}, we can take the maximum value of E, its value at the boundary of the double layer when $x = R_D$. From the curve in Fig. 4 we obtain: $E = 1$ v/cm when $x = R_D$. Then $\tau_{max} = 2 \cdot 10^{-6}$ sec. Let the satellite be a cylinder with length $L_c = 1$ mm. Then its field will be at a given point during the time $\tau_c = \frac{L_c}{v_c} = 10^{-4}$ sec, i.e., $\tau_c \gg \tau_i$, and distribution (1) has time to be established.

The flight time τ_i can also be calculated accurately, by integrating the equation of motion of an ion with potential $\varphi(x)$. For τ_i we obtain the following expression:

$$\tau_i = \int_{x_0}^{0} \frac{dx}{\sqrt{v_{\tau,i}^2 - 2\varepsilon \frac{\varphi(x)}{m}}},$$

where x_0 is the boundary of the double layer, and $v_{\tau,i}$ the thermal velocity of an ion.

If the field E is small then the flight time τ_i is determined by the thermal velocity $\tau_i = d/v_{\tau,i}$.

3. The Double Layer at Equilibrium Ion Concentration

At values of τ_c which are not too high, the change in ion concentration due to their acceleration by the field is not great: it is proportional to the root of the ratio of the potentials (see (11)). When $\varphi_c = -0.77$ v, the ion concentration at the wall $n|_{x=0} = 0.5\ n_o$, where n_o is the ion concentration at the boundary of the layer. At $\varphi_c = -3$ v, $n|_{x=0} = 0.2\ n_o$.

The nonuniformity of the ion distribution n(x) is taken into account in Eq. (12); on the other hand, an error is allowed in determining the boundary conditions, the effect of which on the result is rather difficult to evaluate. Let us attempt to proceed backwards: let us assume that, in general, the ion concentration is constant, which "compresses" somewhat the double layer in comparison with Eq. (12). In this, however, it is possible to satisfy exact boundary conditions: E = 0 not when $x = x_o$ and $\varphi = \varphi_o$, as in (12), but when $x \to \infty$, $\varphi \to 0$. The approximation n= const is especially suitable for the nose surface of a satellite: the velocity of the satellite is greater than the ion thermal velocity and the velocity created by the field, and, therefore, an equilibrium ion distribution does not have time to be established for the given field.

When n_i is not a function of x, the Poisson equation takes the form

$$\frac{d^2\varphi}{dx^2} = 4\pi n_0 \varepsilon\left(e^{\varepsilon\varphi/kT} - 1\right), \qquad (16)$$

or, in dimensionless variables y and ξ, (5)

$$y'' = \frac{1}{2}\left(e^y - 1\right). \qquad (17)$$

Otherwise, it can be written in the form

$$2y''y' = y'e^y - y',$$

i.e.,

$$y' = \sqrt{e^y - y - c}.$$

When $x \to \infty$, $y = y' = 0$, i.e., $c = 1$ and

$$y' = \sqrt{e^y - y - 1}, \tag{18}$$

$$\xi = \int_z \frac{du}{\sqrt{e^u - u - 1}}. \tag{19}$$

This integral expresses φ as a function of x. The curves of $\varphi(x)$, $E(x)$ and $\rho(x)$, calculated by formulas (18) and (19), are shown in Figs. 3, 6 and 10 to 15 (dotted curves).

As was to be expected, the potential decreased more slowly than according to formula (9), the field strength was less, and the thickness of the double layer somewhat greater. For $\varphi = -0.77$ v, the potential decreases to $0.1\varphi_c$ at $x = 5R_D$ (see Fig. 3). An equation similar to (16), but for a spherical problem, has been used by Jastrow and Pearse [2]. We made the following corrections and additions to their work:

1) We went into the conditions of the applicability of Eq. (16) in somewhat more detail.

2) We examined the plane problem and obtained an accurate solution for it (Jastrow and Pearse [2] examined only the spherical problem, the corresponding equation was solved numerically on computers). The double layer can be assumed plane for $d \ll L_c$, i.e., for Sputnik III, Explorer III and other satellites with linear dimensions of 1 meter and larger (d is the thickness of the double layer, and L_c is the linear dimension of the satellite).

3) In accordance with experimental data [8], we took a different electron temperature: $kT_e = 0.15$ ev, lower by a factor of 10, and constructed graphs of the field distribution in the double layer which correspond more accurately ($kT_e = 0.15$ ev) to the experimental characteristics of the ionosphere (the calculations were made at IZMIRAN by G. M. Sosnovskaya and Yu. G. Ishchuk, whom the author thanks).

In conclusion to this section, let us make some remarks concerning concrete conditions for a satellite.

1. The distribution of electrical charges on the surface of a satellite is not, strictly speaking, static. The total ion current over the entire surface of a satellite is equal to the total electron current over its entire surface. But the electron currents on individual sections of the surface are not equal to the ion currents. For example, there is almost no inleakage of ion current to the tail section S_3 (Fig. 16), there are almost no ions in the tail cone, and the current to the satellite is purely electronic. Conversely, ion current predominates on the nose surface S_1. As a result, current flows along the satellite from the nose surface to the tail. As calculation shows, this current is on the order of 10^{-3} a.

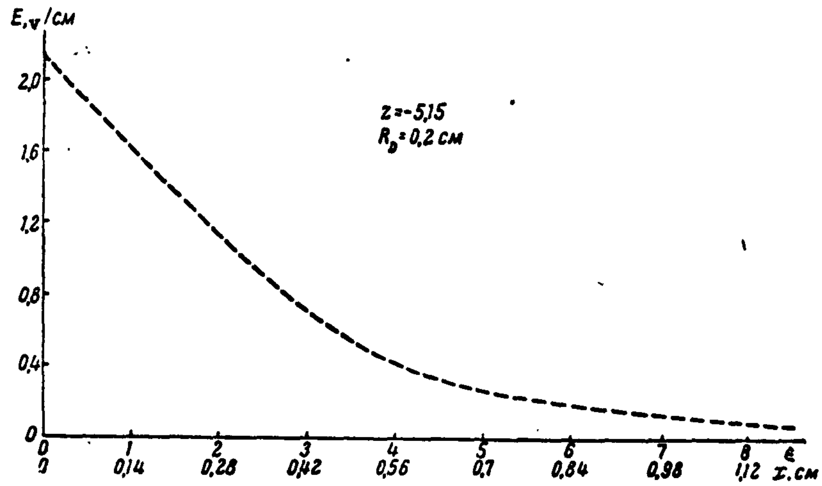


Fig. 10.

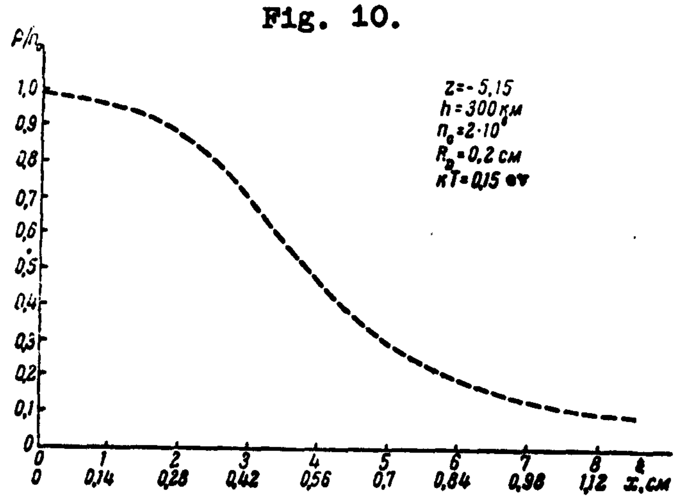


Fig. 11.

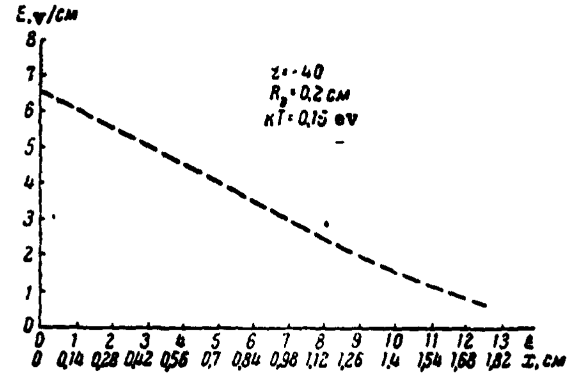


Fig. 12.

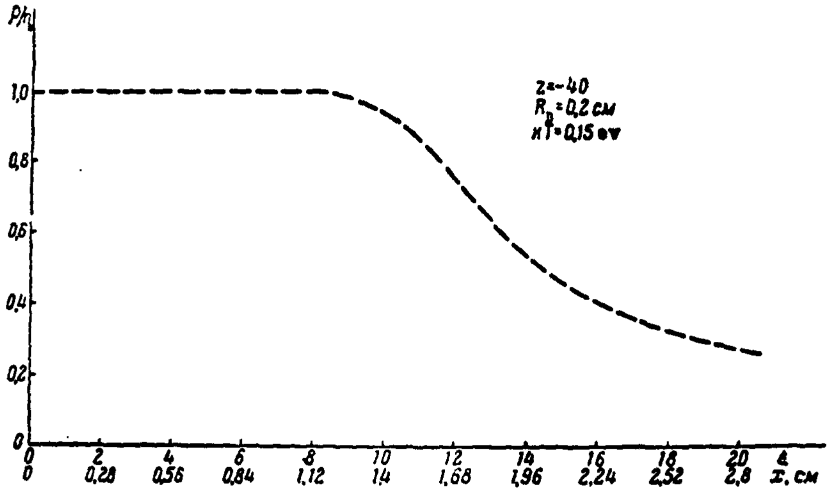


Fig. 13.

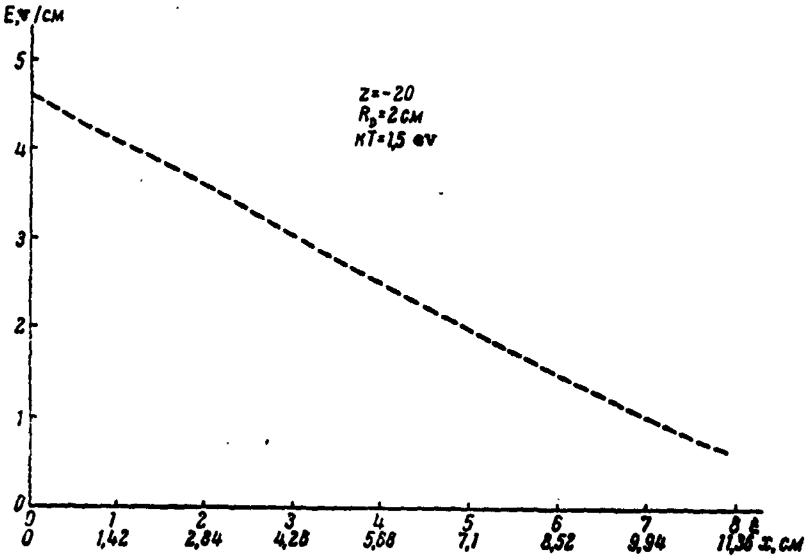


Fig. 14.

Thus the satellite serves as a place for recombination of electrons and ions. In recombination, energy on the order of the ionization energy is liberated. It is very low, but the effect of the liberation by the satellite of ionospheric energy holds in principle*.

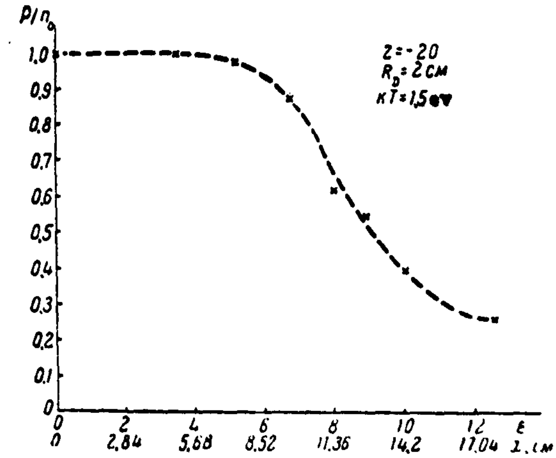


Fig. 15

2. Double layers are formed on any object (antennas, measuring instruments, etc.) issuing from the body of the satellite into the ionosphere. The interaction of two double layers 1 and 2 leads to the formation of a potential well with a depth of up to several volts (Fig. 17 shows the potential curve along the line M_1M_2; A_1 and A_2 are two antennas). By falling into this well, an electron can accomplish oscillations in it and be accelerated to considerable energy (on the order of $\varepsilon\varphi_c$.) This can serve as a source of electron oscillations, and its harmful effect should be borne in mind when putting various measuring instruments into the ionosphere.

3. Let us calculate the capacitance of the double layer. The capacitance of a unit surface of a satellite is found by the expression

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* Of course, ionospheric energy is imparted to the satellite in another way: in collisions of ions, electrons and molecules against the surface of the satellite.

$C = \sigma/\varphi_c$, where the surface density of the charge $\sigma = \frac{E}{2\pi}\Big|_{x=0}$. Let us calculate $y'_{x=0}$ by formula (7):

$$y'_{x=0} = -2\sinh\frac{z}{2} \text{ and } E_{x=0} = -2\sqrt{2}\frac{kT}{\varepsilon}\frac{1}{R_D}\sinh\frac{z}{2}, \tag{20}$$

i.e.,

$$C = \frac{\sqrt{2}}{\pi}\frac{1}{R_D}\frac{\sinh\frac{z}{2}}{z}. \tag{21}$$

The differential capacitance is also an important characteristic of the double layer:

$$C_{dif} = \frac{d\sigma}{d\varphi_c} = \frac{1}{2\pi\sqrt{2}}\frac{1}{R_D}\cosh\frac{z}{2}. \tag{22}$$

As distinct from the capacitance of linear systems, C and C_{dif} are functions of φ_c; the charge is not proportional to the potential. Similarly, from Eq. (18) we find

$$\begin{aligned} E_{x=0} &= \sqrt{2}\frac{kT}{\varepsilon}\frac{1}{R_D}\sqrt{e^z - z - 1}, \\ C &= \frac{1}{\pi\sqrt{2}}\frac{1}{zR_D}\sqrt{e^z - z - 1}, \\ C_{dif} &= \frac{1}{2\pi\sqrt{2}}\frac{1}{R_D}\frac{e^z - 1}{\sqrt{e^z - z - 1}}. \end{aligned} \tag{23}$$

Measurement of the capacitance of the double layer (for example, the alternating current differential capacitance) would make it possible to determine the true structure of the double layer. The simultaneous measurement of the potential φ_c and the field strength at the wall would be a way to check the theory of the double layer.

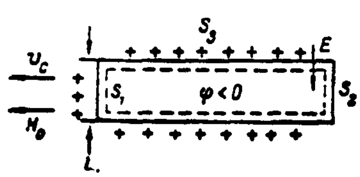

Fig. 16

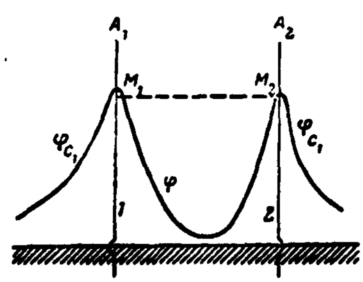

Fig. 17

4. Under the influence of intensive radiation in outer space, the surface of a satellite can change its properties of reflection and neutralization of ions striking it, which, in turn, would change the value of the potential φ_c and the structure of the double layer.

4. The Influence of the Magnetic Field

The geomagnetic field does not affect the Maxwell-Boltzmann distribution (1). The kinetic equation in the presence of an external magnetic field H has the form

$$\frac{\partial f}{\partial t} + u \operatorname{grad}_r f + \left(\frac{e}{m} E + \frac{e}{c}[uH]\right) \operatorname{grad}_u f = 0. \tag{24}$$

As is easily seen by substitution, distribution (1) also satisfies this equation. It reduces to zero not only the terms $u \operatorname{grad}_r f + \frac{e}{m} E \operatorname{grad}_u f$, but also the magnetic term $[uH]\frac{df}{\partial u} = H_0\left[u_y \frac{\partial f}{\partial u_x} - u_x \frac{\partial f}{\partial u_f}\right]$. Therefore, the structure of steady-state double layer (4) is not a function of H, although, of course, the electron trajectories will be completely different. Conversely, the set-up time of the double layer is essentially a function of H: charged particles cannot move freely across the field H, and the presence of diffusion across the field is due, in Bohm's opinion [7], to a rather complex mechanism, plasma oscillations (fluctuations), which still has not been explained definitely (Chapter 2 [7]).

The Larmor radius of a particle $R_H = cmu_T/eH$ is a characteristic which determines the influence of the magnetic field. In our concrete boundary-value problem it is the magnitude of the Larmor radius relative to the other characteristic parameters of length: the dimensions of the satellite L_c and the thickness of the double layer $\underline{d}$.

For ions, R_H = 3 to 4 m, i.e., it is of the same order of magnitude as the dimensions of large satellites. Therefore, the magnetic field influences the ion current to the satellite. The fact of the matter is that the approximation of the Langmuir-Mott-Smith probe theory [4, 9] which is usually used [1, 2] for calculating the current to a satellite and the potential φ_c is valid only when the dimensions of the probe (in our case a satellite) are many times smaller than the path length λ_1. When $H = 0, \lambda_1 \sim 10^5$ cm and $\lambda_1 \gg L_c$; this condition of the applicability of the probe theory is fulfilled. When $H \neq 0$, the role of the "lower boundary" of λ_1 for the path across the field is played by the Larmor radius R_H. Inasmuch as $R_H \sim L_c$, it must be taken into account that the presence of a probe (satellite) disturbs the plasma, changing the number of particles in the surrounding space. The magnetic field affects electron motion to an even greater degree. For electrons, the Larmor radius $r_H \sim 2$ cm, i.e., the inverse inequality $r_H \gg L_c$ is fulfilled and the geomagnetic field considerably decreases the electron current to the probe.

The magnetic field also affects the trajectory of particles reflected from the surface of the satellite. In flight along the field H, electrons reflected from the nose surface of a satellite move along the lines of force, forming an electron beam. The direction of this motion (along the lines of force of the field H) is created by the magnetic field. There will be an ion current in front of the satellite. As with the electrons, the ions will move along the lines of force. The distance at which these beams are still "distinguishable" in front of the satellite is on the order of that of the free path along the field, $\lambda \sim 10^5$ cm.

In flight across the field H, the reflected beam flies in front of the satellite at a considerably shorter distance. It is on the order of r_H (Fig. 16), because an electron moves simultaneously along both the Larmor orbit and the field, i.e., along the nose surface of the satellite. The beam will not be as intensive in flight perpendicular to the field as it is in longitudinal flight. The most interesting effect connected with the magnetic field is, perhaps, electrical drift. As is known [10, 11], in the presence of a magnetic field H and an electrical field E simultaneously, a charged particle acquires, besides Larmor precession, an additional motion in a direction perpendicular to both H and E with velocity

$$W_D = \frac{c\,[\mathrm{EH}]}{H^2}. \tag{25}$$

This motion is called electrical drift. The electrical field E in our case is the field of the double layer, the magnetic field H is the geomagnetic field.

Unfortunately, the division of motion into Larmor precession and uniform drift (25) is valid only in a sufficiently uniform electric field, where $r_H << d$. In our case $r_H \sim d$, therefore, the drift velocity should be determined not by Eq. (25), but from accurate equations of motion of an electron $m\dot{\mathbf{r}} = -\varepsilon\mathbf{E} - \frac{\varepsilon}{c}[\mathbf{uH}]$. The solution of these equations has the form

$$t = \int_{x_0}^{x} \frac{dx}{\sqrt{\frac{1}{m}[W_0 - \varepsilon\varphi(x)] + [\dot{y}_0 + \omega_H(x - x_0)]^2}},$$

where W_0 and $\dot{y}_0$ are the total energy and velocity of an electron in the direction of the drift, $\varphi(x)$ the potential of the electric field, ω_H the Larmor frequency, x_0 the distance of an electron from the wall at the initial moment. For the other coordinate $\underline{y}$ we have

$$\dot{y} = \dot{y}_0 + \omega_H(x - x_0).$$

In flight along the field (Fig. 18 a and b), drift leads to flow around the satellite in the plane perpendicular to the flight. Electrons will move relative to the satellite in a spiral. In flight perpendicular to the field (Fig. 19), drift flow-around will occur in the longitudinal plane.

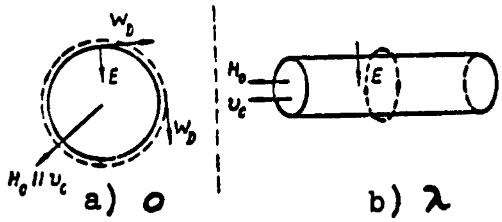

Fig. 18

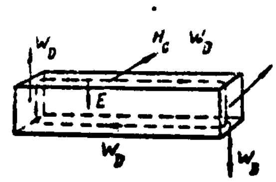

Fig. 19

A natural question arises: with a negatively charged satellite, there are very few electrons near its wall ($n_e \approx n_o e^{\frac{\varepsilon\varphi}{kT}}$; Figs. 2, 7, 12, 15), while at the place where their concentration is close to n_o, the electric field is zero, $W_D = 0$, and it would appear that there is no drift. But this is not the case at all. Even near the boundary of the double layer, when $n_e \sim 10^{-2}n_o$, $E \sim 10^{-1}$ v/cm and W_D, according to formula (25), is on the order of 10^7 cm/sec (for example, on the curve for φ in Fig. 13, when $x = 5R_D = 5$ cm, $\varphi_c = -3$ v, $E = 0.15$ v/cm and $W_D \sim 10^7$ cm/sec). Of course, formula (25) is not, strictly speaking, valid for $r_H \sim d$, and the drift velocity is really somewhat less, but here we wish only to indicate the existence of such an effect (electrical drift in the field of the double layer of a satellite) and not calculate its value.

For the effect of drift flow-around, satellite motion is not at all compulsory (although, of course, it exerts an influence). Any body placed in a plasma acquires a surface charge, and drift about it arises from the influence of the magnetic field. The essential singularity of a satellite is a different matter: its dimensions are much less than the path length, and drift (in the ionosphere) is not complicated by collisions (in any case, in the first approximation).

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